VPM-B algorithm

Jan Darowski

August 21, 2015

Abstract

This article is a description of the VPM-B decompression algorithm as implemented for the Subsurface project, during the Google Summer of Code 2015 program. Its main goal is to explain next phases of the algorithm and an intuitive meaning of calculations. It assumes some basic understanding of the Bubble Theory and does not provide full derivations of all the equations (however, they can be found in the pointed resources).

1 The Algorithm

First part of the VPM-B algorithm is a *dive simulation*. It starts from the begining of the dive and is intended to gather all the information about a state of the diver's body in the moment of the start of ascent. This includes *He and N_2 saturation* in tissue compartments and the maximum *crushing pressure* for each compartment. These two variables are the main factors, that decide how fast the diver can ascend at the end of the dive.

After the dive simulation, a *start gradient* is calculated. It is the maximum allowable difference between the *ambient pressure* and the *tissue saturation*. This gradient is calculated so that the maximum number of bubbles in each tissue is not exceeded after the ascent. Then, the *Critical Volume Algorithm* starts. It simulates the ascent with the calculated gradient, checks if the gradient can be improved (increased) and starts again with a new gradient. It keeps the volume of the generated gas below some constant safe level. Next gradients converge pretty fast, so after a few iterations, the final deco profile is set. For more conservative results, during the ascent simulation, Boyle's law compensation is added. It modifies the gradient on every deco stop, to represent the bubbles' expansion during the decompression.

2 Critical radius

Basic VPM assumption is, that in every tissue compartment, some safe number of nuclei can grow into bubbles. Ordering Theory says, that when applying some pressure schedule on a group of nuclei, their size order is kept. So we can introduce some gas-specific critical radius of a nucleon, that is the biggest one that can grow into a bubble. Then, we can simulate the pressure schedule on this nucleon and we will be able to say, what is its final size, just prior the ascent. Knowing that, the only thing we need to do, is prevent it from growing into a bubble.

For VPM-B, 0.45 and 0.55 microns are proposed as the basic values for He and N_2 nuclei respectively. This values can be increased for more conservative deco.

3 Dive Simulation

3.1 Tissue saturation

In the Bubble Theory based algorithms, saturation of He and N_2 inside 16 different tissue compartment types is calculated. In this paper I will call it P_{sat}. This value can be easily calculated using Schreiner's and Haldane's equations. They are quiet popular so I will not describe them here. Haldane equation assumes, that the diver stays at a constant depth. Schreiner equation is its expansion, for a linear ascent / descent.

The original code used both of these equations, but as the Subsurface implementation already had the tissue saturation calculation using only the Haldane's equation on 1s segments, I kept it this way, checking that it does not have any influence on the results.

3.2 Crushing pressure

In normal, permeable situation, there is a direct relation between the ambient pressure (P_{amb}) and the pressure of gas in a nucleon (P_{in}), inside the tissue. In such a situation, we define the crushing pressure as the difference between the ambient pressure and the total tissue saturation:

$$H = P_{crush} = P_{amb} - (P_{sat_{He}} + P_{sat_{N_2}} + P_{sat_{other}})$$

And in this situation, crushing pressure for both gases is the same.

But the nucleon is not always permeable! The H value can be understood as a pressure gradient on the walls of the nucleon. If it's too big, the bubble

stops being permeable, gases cannot freely flow in and out the nucleon and $P_{crush} = H$ is no longer kept. VPM-B authors assume some constant value for all the compartments, that defines the critical gradient of permeability H_{max}. So what if $H > H_{max}$? Then, for the given nucleon, knowing its radius at the beginning of the dive, we can calculate its radius in the moment of becoming impermeable. Next, using the Bubble Theory equations, we can calculate its radius in any moment of the impermeable phase, knowing the ambient pressure. From there, we can calculate the pressure inside the nucleon using the Boyle's law.

$$r_{onset} = \frac{1}{\frac{H_{max}}{2 \cdot (\gamma_c - \gamma)} + \frac{1}{r_{start}}}$$

$$A = P_{amb} - H_{max} + \frac{2 \cdot (\gamma_c - \gamma)}{r_{onset}}$$

$$B = 2 \cdot (\gamma_c - \gamma)$$

$$C = P_{sat_{onset}} \cdot {r_{onset}}^3$$

$$A \cdot {r_{curr}}^3 - B \cdot {r_{curr}}^2 - C = 0$$

$$P_{in} = \frac{P_{sat_{onset}} \cdot {r_{onset}}^3}{{r_{curr}}^3}$$

$$P_{crush} = P_{amb} - P_{in}$$

where

r_{onset}	nucleon's radius in the last permeable moment
r_{start}	nucleon's radius at the beginning of the dive
γ	surface tension constant
γ_c	skin compression constant
P_{amb}	current ambient pressure
$P_{sat_{onset}}$	compartment's saturation in the last permeable moment
r_{curr}	current nucleon's radius

3.3 Bottom time

When the profile reaches the end of the bottom time, we can calculate radius of the nuclei starting with the specified *Critical radius*, after applying the crushing pressure on them. Then, authors of the algorithm suggest nuclear

regeneration calculations, but their influence on the results is minimal.

$$r = \frac{1}{\frac{P_{crush}}{2 \cdot (\gamma_c - \gamma)} + \frac{1}{r_{start}}}$$
$$r_{regen} = r + (r_{start} - r) \cdot (1 - e^{\frac{-t}{T}})$$

where

r	nucleon's radius after the crushing pressure
r_{start}	initial nucleon's radius
r_{regen}	radius after the nuclear regeneration
t	time from the start of the dive
T	time regeneration constant

4 Start Gradient

When we are at the end of the bottom time, we need to find out what conditions would make our tracked critical nucleon grow into a bubble. Having the radius of the nucleon, we can calculate, what gradient between the tissue saturation and ambient pressure would make it grow into bubble (G_{start}). Then, we can simulate the ascent, keeping an eye on the current gradient (G), stopping when the calculated max gradient would be violated. For the single nucleon, the G_{start} can be formulated as follows:

$$G_{max} = G_{start} = 2 \cdot \frac{\gamma}{\gamma_c} \cdot \frac{\gamma_c - \gamma}{r}$$

where

r	radius of the nucleon prior the ascent

We need to calculate it for both He and N_2 separately. Then, the total gradient for a compartment is given by:

$$G_{start} = \frac{G_{start_{He}} \cdot P_{sat_{He}} + G_{start_{N_2}} \cdot P_{sat_{N_2}}}{P_{sat_{He}} + \cdot P_{sat_{N_2}}}$$

For the given compartment, G is defined as follows:

$$G = P_{sat_{He}} + P_{sat_{N_2}} + P_{sat_{other}} - P_{amb}$$

where
$P_{sat_{other}}$ other tissue gases constant

5 Ascent Simulation

During the ascent simulation, we just need to keep track of the current G value and make a deco stop when it gets too close to G_{max}. That's the theory. Original code by Baker makes something slightly different. At the end of each deco stop, it waits, until it could reach the next stop in 0 time, without violating G_{max}. It makes the schedule a little bit more conservative than if we take the ascent time into account. Especially for dives with a short bottom time. In Subsurface, the original idea has been implemented.

5.1 Boyle's Law Compensation

The letter B in VPM-B states for the Boyle's Law Compensation, which is a small modification of the original algorithm, that greatly increases deco time of deeper dives. The idea is, that because of a depth difference during the ascent, nuclei grow. So the gradient needed to turn the initial nucleon into bubble decreases. To implement this idea, we assume, that the G_{start} was the gradient working fine only to the first deco stop. Then, on every stop, we update the current G_{max}, so it works fine for the next deco stop.
What is important from the practical point of view: also gas change stops should be treated as deco stops. If the first gas change is before any other

deco stop, all the compensation should be done in relation to it.

$$
\begin{aligned}
r_{first} &= \frac{2 \cdot \gamma}{G_{bottom}} \\
A &= P_{amb_{next}} \\
B &= -2 \cdot \gamma \\
C &= (P_{amb_{first}} + \frac{2 \cdot \gamma}{r_{first}}) \cdot {r_{first}}^3 \\
A \cdot {r_{next}}^3 - B \cdot {r_{next}}^2 - C &= 0 \\
G_{max_{new}} &= \frac{2 \cdot \gamma}{r_{next}}
\end{aligned}
$$

where

r_{first}	radius of nucleon at the first stop
G_{bottom}	gradient for the first stop (G_{start} or from CVA)
$P_{amb_{next}}$	ambient pressure at the next stop
$P_{amb_{first}}$	ambient pressure at the first stop

6 Critical Volume Algorithm

The basic VPM algorithm, without CVA, assumes that the creation of bubbles from nuclei is instant and body can tolerate some number (N_{safe}) of them forever. But in real dives, it is possible to accept some more bubbles, as long as they do not exceed the critical volume (V_{crit}) of free gas generated during the desaturation. The free gas volume is assumed to be proportional to the integral of G in time, during the desaturation. To simplify calculations, We assume, that through the whole ascent $G = G_{max}$. And so we can easily calculate $G \cdot t_{deco}$. But the free gas is also generated at the surface, where the gradient decays exponentially.

CVA's goal is to generate a new $G_{max_{new}}$, that will be closer to generating V_{crit} free gas. It is not precise and depends on the deco time, so we must simulate the whole ascent several times, recalculating G_{max}, recording t_{deco} and checking if it still differs significantly from the previous time. The

formula for the next $G_{max_{new}}$ is as follows:

$$B = G_{start} + \frac{\lambda \cdot \gamma}{\gamma_c \cdot t_{desat}}$$
$$C = \frac{\gamma^2 \cdot \lambda \cdot P_{crush}}{{\gamma_c}^2 \cdot t_{desat}}$$
$$G_{max} = \frac{B + \sqrt{B^2 - 4C}}{2}$$

where

λ	critical volume $\times$ time constant

Of course it needs to be calculated for all 16 compartments, for two gases and the final result is a weighted average of $G_{max_{He}}$ and $G_{max_{N_2}}$. Just like in the case of G_{start}. What can be unclear here is the t_{desat}. It is the total time of desaturation, assuming that it is done with $G = G_{max_{old}}$, also on the surface!

$$t_{surf1} = \frac{\frac{P_{sat_{He}}}{T_{He}} + \frac{P_{sat_{N_2}} - P_{N_2Ins}}{T_{N_2}}}{P_{sat_{He}} + P_{sat_{N_2}} - P_{N_2Ins}}$$

$$T_G = \frac{ln(\frac{P_{N_2Ins} - P_{sat_{N_2}}}{P_{sat_{He}}})}{T_{N_2} - T_{He}}$$
$$VT_G = \frac{P_{sat_{He}}}{T_{He}} \cdot (1 - \exp(-T_{He} \cdot T_G)) + \frac{P_{sat_{N_2}} - P_{N_2Ins}}{T_{N_2}} \cdot (1 - \exp(-T_{N_2} \cdot T_G))$$
$$t_{surf2} = \frac{VT_G}{P_{sat_{He}} + P_{sat_{N_2}} - P_{N_2Ins}}$$

$$t_{desat} = t_{deco} + t_{surf}$$

where

t_{surf1}	when $P_{sat_{N_2}} > P_{N_2Ins}$
t_{surf2}	otherwise
P_{N_2Ins}	inspired N_2 pressure at surface
T_x	halflife of x

7 Conservatism levels

As the basic parameterization is prepared for very aggressive divers, additional conservatism levels are implemented. Next levels scale the starting critical radii by $1.05, 1.12, 1.22, 1.35$. These values were proposed by the V-Planner's authors and provide enough spread for most users.

8 Parameters

r_{start}	N_2 critical radius	- 0.55 microns
r_{start}	He critical radius	- 0.45 microns
λ	Critical volume $\times$ time	- 199.58 bar $\cdot$ min
H_{max}	Max gradient of impermeability	- 8.2 bar
γ	Surface tension	- 0.0179 N/m
γ_c	Skin compression	- 0.257 N/m
T	Nucleon regeneration time	- 20160 min
$P_{sat_{other}}$	Other gases saturation	- 0.1359888 bar

9 Resources

1. **http://www.deepocean.net** - Very good explanation of the Bubble Theory and the breathing equations. It is a good place to start, not necessarily to base implementation on, as there are a few mistakes.

2. **https://github.com/bwaite/vpmb** - Rewritten original implementation of VPM-B in C and Python. Very handful for debugging and comparing partial results.

3. **http://www.decompression.org/maiken/VPM/VPM_Program_Site_Map.htm** - Site of the authors of VPM-B. Links to the ftp server containing priceless materials, like CVA equation's derivation or the original code.